Written examination
TIN172/DIT410, Artificial Intelligence

Tuesday 2nd June 2015, 14:00–18:00

Examiner: Peter Ljunglöf, 0736–24 24 76
(will come to the exam at 15:00–15:30)

This examination consists of eight questions of which two are advanced (questions 5 and 8). A correctly answered question gives you one point, and if the answer is almost correct you might get a half point.

Grades To get grade 3/G you need at least 4 points.

To get grade 4/VG you need at least 5 points,
of which at least 1 must be from the advanced questions.

To get grade 5/VG+ you need at least 6 points,
of which at least 2 must be from the advanced questions.

Update! Apparently the advanced questions were very advanced, especially the final one. To compensate for this, I have changed the grading like this:

To get grade 4/VG you need 5 points (including 1 advanced), as explained above, or 6 points (without advanced points).

To get grade 5/VG+ you need 6 points (including 2 advanced), as explained above, or 7 points (including 1 advanced).

Tools Paper and pencil.

Crayons, paper glue, scissors.

One A4 cheat sheet with any information you want on it (both sides allowed).

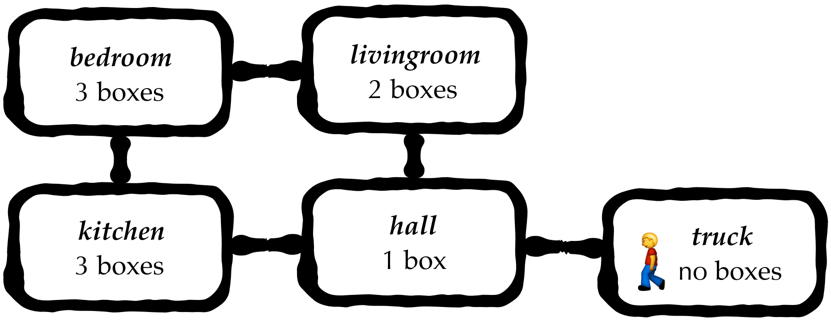
No books or calculators.

Notes Start every question on a new paper,
and write your ID number at the top of every paper.

Write readable, and explain your answers!

Moving out of your apartment

After having successfully completed the AI course, you get a job in another town (as an AI expert of course), and you have to move out of your apartment. You have already packed all your belongings in nine moving boxes, scattered around the apartment, now all you have left to do is to move these boxes into the moving truck, which is parked just outside of the apartment. The boxes are too big to carry, so you have to push them between the rooms. The following graph shows the four rooms in the apartment, the truck, how many boxes are initially in which rooms, and how the rooms are connected:



On each turn, you can either ***move*** or ***push a box*** into an adjacent location, in any direction: ***north, south, east, west***. When this story begins, you have just parked the truck.

Many noticed that "push a box" is unclear: Does the agent move with the box or not? Since it's unclear I told the ones who asked that they should decide that for themselves. In the solution suggestions below I assume that the agent moves together with the box.

Question 1. Formulate this problem as a search problem, i.e.:

a) What is a suitable representation of the search states?

A tuple *[agent, bedroom, livingroom, kitchen, hall, truck]*, where $agent \in \{b,l,k,h,t\}$ is the location of the agent, and the others are numbers (≥0) telling how many boxes are in that respective room.

Note that you can skip one of the room numbers, since the total number of boxes is fixed.

b) How will the starting state look like?

[truck, 3, 2, 3, 1, 0]

c) How will the goal check be?

E.g., *[?, ?, ?, ?, ?, 9]*, or $state[5] == 9$, or something like that.

Question 2. The following is a heuristic, where $N(r)$ is the number of boxes in location r:

$$h = \sum_{r \neq truck} N(r)$$

a) Explain why h is admissible.

Every box must be moved into the truck, and it will take at least one push action for each box.

b) Describe a different admissible heuristics h' that dominates h (i.e., $h' \geq h$ in all search states). Explain why it is admissible and why it is dominating.

Alternative 1: After pushing a box into the truck, the agent must go back into the house, which takes at least one extra action. So, $h' = 2h$ is also admissible, and obviously dominating.

Alternative 2: A box in the livingroom or kitchen takes at least two pushes, and a bedroom box takes at least three moves. So, $h' = h + 2N(bedroom) + N(livingroom) + N(kitchen)$ is admissible and dominating.

Alternative 3: We can even combine the alternatives. This is left as an exercise for the reader.

Question 3. Formulate the problem as a STRIPS planning problem, i.e.:

a) What features variables are needed, and what are their respective domains?

This is very similar to question 1a: The variables are *agent, bedroom, livingroom, kitchen, hall* and *truck*, and their domains are as in 1a.

b) What actions are there?

In pure STRIPS, the actions don't have arguments, so there will be 20 different actions: *MoveFromAtoB* and *PushFromAtoB*, where *A* and *B* are all legal combinations of rooms.

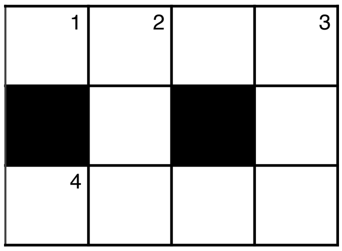
Examples: *MoveFromTruckToHall, MoveFromHallToTruck, PushFromHallToKitchen*, etc.

c) Give a STRIPS definition of one of the pushing actions.

<u>PushFromHallToKitchen</u>

Preconditions: $agent = h \ \wedge \ hall > 0$
Effects: $agent' = k \ \wedge \ hall' = hall{-}1 \ \wedge \ kitchen' = kitchen{+}1$

Crossword puzzle



Lexicon: APA, ASP, FALK, FALL, KAP, KUL, SPEL

This crossword puzzle can be formulated as a CSP using the following variables and constraints:

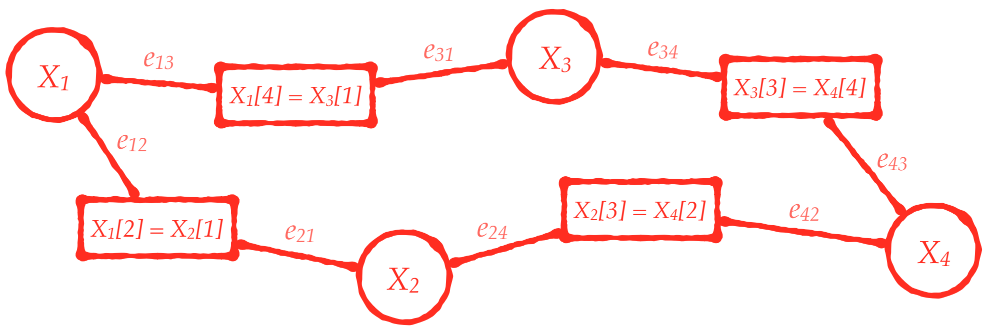
Variables are of the form X_i, meaning to place a vertical or horizontal word in the ith square. All variables have the whole lexicon as domain.

> Many noticed that this was unclear. The meaning is that there should be four variables, where X_1 and X_4 correspond to horisontal 4-letter words and X_2 and X_3 are vertical 3-letter words.

Constraints are of the form $X_i[n] = X_j[m]$, meaning that the nth letter of word X_i is equal to the mth letter of word X_j; and of the form $len(X_i) = k$, meaning that the word X_i has k letters.

Question 4. Draw the constraint graph and make it arc consistent, i.e.:

a) Perform domain consistency and draw the constraint graph with the resulting variable domains. Name the arcs so that you can refer to them in (b).



> After domain consistency, the domains are
>
> $X_1, X_4 \in \{FALK, FALL, SPEL\}$
> $X_2, X_3 \in \{APA, ASP, KAP, KUL\}$
>
> When the CSP is domain consistent, the unary constraints $len(X_i)=k$ can be removed, so we don't have to show them in the constraint graph.

b) Perform arc consistency on the whole graph. Write in which order you process the arcs.

See this table. Note that this is only one possibility out of many.

arc	variable	remove	domain	reinsert	arcs-to-do
e_{12}	X_1	SPEL	FALK, FALL	(e_{31})	e_{21}, e_{13}, **e_{31}**, e_{24}, e_{42}, e_{34}, e_{43}
e_{21}	X_2	KAP, KUL	APA, ASP	(e_{42})	e_{13}, e_{31}, e_{24}, **e_{42}**, e_{34}, e_{43}
e_{13}	X_1	FALL	FALK	e_{21}	e_{31}, e_{24}, e_{42}, e_{34}, e_{43}, **e_{21}**
e_{31}	X_3	APA, ASP	KAP, KUL	(e_{43})	e_{24}, e_{42}, e_{34}, **e_{43}**, e_{21}
e_{24}	X_2	—	APA, ASP	—	e_{42}, e_{34}, e_{43}, e_{21}, e_{13}
e_{42}	X_4	—	SPEL, FALK, FALL	—	e_{34}, e_{43}, e_{21}, e_{13}
e_{34}	X_3	KAP	KUL	(e_{13})	e_{43}, e_{21}, **e_{13}**
e_{43}	X_4	FALK	SPEL, FALL	e_{24}	e_{21}, e_{13}, **e_{24}**
e_{21}	X_2	—	APA, ASP	—	e_{13}, e_{24}
e_{13}	X_1	—	FALK	—	e_{24}
e_{34}	X_3	—	KUL	—	—

c) How will the domains be after the whole graph is arc consistent?

After arc consistency, the domains are

$X_1 \in$ {FALK}
$X_2 \in$ {APA, ASP}
$X_3 \in$ {KUL}
$X_4 \in$ {SPEL, FALL}

Question 5* (advanced). Perform Variable Elimination on this CSP:

This can be done on the original (domain consistent) constraint graph, or the arc consistent. Depending on which you get different results. Below I perform VE on the arc consistent graph.

a) First eliminate the variable corresponding to the rightmost vertical word.

The rightmost vertical word corresponds to variable X_3, which occurs in two constraints, C_{13}: $X_1[4]=X_3[1]$ and C_{34}: $X_3[3]=X_4[4]$, so we merge them into one new constraint R_{14} over X_1 and X_4:

C_{13}

X_1	X_3
FALK	KUL

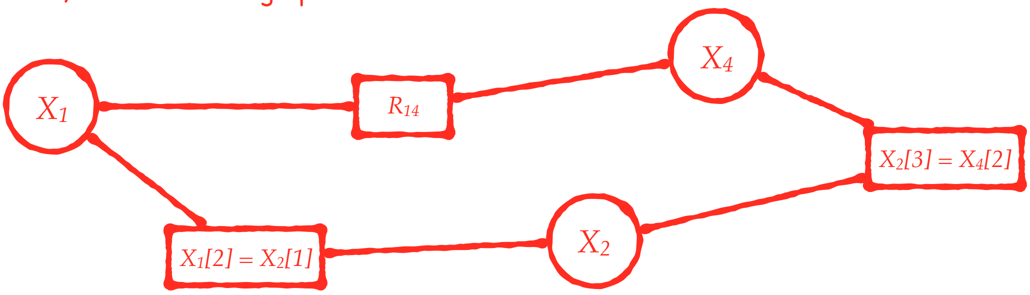
C_{34}

X_3	X_4
KUL	FALL
KUL	SPEL

R_{14}

X_1	X_4
FALK	FALL
FALK	SPEL

After this, the constraint graph looks like this:



b) Then eliminate the bottommost horizontal word.

This word corresponds to variable X_4, which occurs in two constraints, C_{24}: $X_2[3]=X_4[2]$ and R_{14} from above, so we merge then into one new constraint R_{12} over X_1 and X_2:

R_{14}

X_1	X_4
FALK	FALL
FALK	SPEL

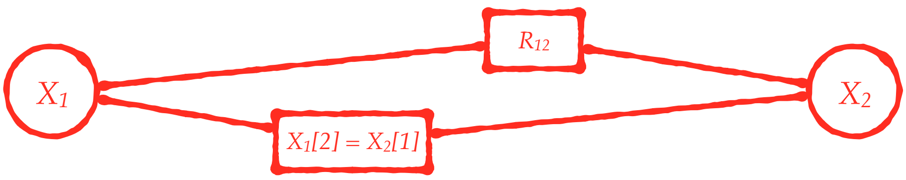
C_{24}

X_4	X_2
FALL	APA
SPEL	ASP

R_{12}

X_1	X_2
FALK	APA
FALK	ASP

In the end the CSP has two variables, X_1 and X_2, and two constraints, R_{12} and C_{12}: $X_1[2]=X_2[1]$:



Ragnarök

Rising ocean levels (O) can be a result of global warming (G), or be a sign that Ragnarök (R) has started, the final apocalypse in Norse mythology. Another result of Ragnarök is that the sun is devoured (D) by the Fenris wolf.

The probability that Ragnarök has started is 10%, but global warming is happening with 90% probability. If Ragnarök is happening, then it is absolutely certain that the oceans will rise and the sun will be devoured. However, if Ragnarök is not happening, we have the following probabilities:

$$P(+d \mid -r) = 20\%$$
$$P(+o \mid -r, +g) = 80\%$$
$$P(+o \mid -r, -g) = 40\%$$

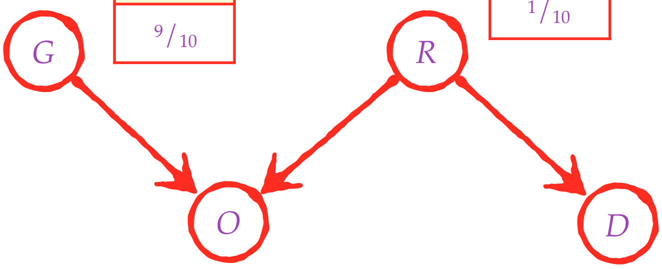
(where $+x$ means that the event X is a fact, and $-x$ means that X is false).

Question 6. Draw the Bayesian network corresponding to the information given above.

Also write down the corresponding probability tables.

R	G	$P(+o \mid R,G)$
$+r$	$+g$	1
$+r$	$-g$	1
$-r$	$+g$	$4/5$
$-r$	$-g$	$2/5$

$P(+g)$
$9/10$



$P(+r)$
$1/10$

R	$P(+d \mid R)$
$+r$	1
$-r$	$1/5$

Question 7. Calculate the following probabilities:

a) What is the probability that the sun is devoured?

$$P(+d) = P(+d, +r) + P(+d, -r)$$
$$= P(+d \mid +r)\,P(+r) + P(+d \mid -r)\,P(-r)$$
$$= 1 \cdot 1/10 + 1/5 \cdot 9/10 = 1/10 + 9/50 = 28/100 = 7/25 = 28\%$$

b) What is the probability that Ragnarök is happening, given that the sun is devoured?

Using Bayes' theorem:

$$P(+r \mid +d) = P(+d \mid +r)\,P(+r) \;/\; P(+d)$$
$$= 1 \cdot (1/10) \;/\; (7/25) = 100/280 = 5/14 \approx 35.7\%$$

Since you don't have a calculator, you don't have to do the calculations – it's enough if you answer as a quotient (e.g., 34/86). But you have to explain how you got that result.

Question 8* (advanced). Calculate the following probabilities using Variable Elimination:

a) What is the probability of Ragnarök, given that the oceans are rising?

So, we have to calculate the conditional probability $P(+r \mid +o)$ using VE.
First, the Bayesian network has the following assumption:

$$P(G,R,O,D) = P(O \mid R,G)\, P(D \mid R)\, P(G)\, P(R)$$

Now we use this to calculate the factors $f(R)$ and $g(R)$:

$$\begin{aligned}
f(R) &= P(R,+o) \\
&= \textstyle\sum_{G,D} P(G,R,+o,D) \\
&= \textstyle\sum_{G,D} P(+o \mid R,G)\, P(D \mid R)\, P(G)\, P(R) \\
&= P(R) \textstyle\sum_G P(+o \mid R,G)\, P(G) \textstyle\sum_D P(D \mid R) \\
&= P(R) \textstyle\sum_G P(+o \mid R,G)\, P(G) \qquad \text{[because } \textstyle\sum_D P(D \mid R) = 1 \text{ by definition]}
\end{aligned}$$

$$\begin{aligned}
g(R) &= \textstyle\sum_G P(+o \mid R,G)\, P(G) \\
&= \{ +r\!: P(+o \mid +r,+g)\, P(+g) + P(+o \mid +r,-g)\, P(-g)\, ;\ -r\!: P(+o \mid -r,+g)\, P(+g) + P(+o \mid -r,-g)\, P(-g) \} \\
&= \{ +r\!: 1 \cdot {}^{9}/_{10} + 1 \cdot {}^{1}/_{10} = 1\, ;\ -r\!: {}^{8}/_{10} \cdot {}^{9}/_{10} + {}^{4}/_{10} \cdot {}^{1}/_{10} = {}^{76}/_{100} = {}^{19}/_{25} \}
\end{aligned}$$

$$\begin{aligned}
f(R) &= P(R)\, g(R) = \{ +r\!: P(+r)\, g(+r)\, ;\ -r\!: P(-r)\, g(-r) \} \\
&= \{ +r\!: {}^{1}/_{10} \cdot 1 = {}^{1}/_{10}\, ;\ -r\!: {}^{9}/_{10} \cdot {}^{19}/_{25} = {}^{684}/_{1000} = {}^{171}/_{250} \}
\end{aligned}$$

Finally we can use $f(R)$ for calculating $P(+r \mid +o)$:

$$\begin{aligned}
P(+r \mid +o) &= f(+r) \,/\, \textstyle\sum_R f(R) = ({}^{1}/_{10}) \,/\, ({}^{1}/_{10} + {}^{171}/_{250}) = ({}^{1}/_{10}) \,/\, ({}^{196}/_{250}) \\
&= {}^{100}/_{784} = {}^{25}/_{196} \approx 12.8\%
\end{aligned}$$

b) What is the probability of Ragnarök, given that the oceans are rising and that the sun is devoured?

This time we want to calculate the conditional probability $P(+r \mid +o,+d)$ using VE.
For this we need the factor $h(R)$:

$$\begin{aligned}
h(R) &= P(R,+o,+d) \\
&= \textstyle\sum_G P(G,R,+o,+d) \\
&= \textstyle\sum_G P(+o \mid R,G)\, P(+d \mid R)\, P(G)\, P(R) \\
&= P(R)\, P(+d \mid R) \textstyle\sum_G P(+o \mid R,G)\, P(G) \\
&= P(R)\, P(+d \mid R)\, g(R) \\
&= \{ +r\!: P(+r)\, P(+d \mid +r)\, g(+r)\, ;\ -r\!: P(-r)\, P(+d \mid -r)\, g(-r) \} \\
&= \{ +r\!: {}^{1}/_{10} \cdot 1 \cdot 1 = {}^{1}/_{10}\, ;\ -r\!: {}^{9}/_{10} \cdot {}^{1}/_{5} \cdot {}^{19}/_{25} = {}^{1368}/_{10000} = {}^{171}/_{1250} \}
\end{aligned}$$

Now we can use $h(R)$ for calculating $P(+r \mid +o,+g)$:

$$\begin{aligned}
P(+r \mid +o,+g) &= h(+r) \,/\, \textstyle\sum_R h(R) = ({}^{1}/_{10}) \,/\, ({}^{1}/_{10} + {}^{171}/_{1250}) = ({}^{1}/_{10}) \,/\, ({}^{296}/_{1250}) \\
&= {}^{1000}/_{2368} = {}^{125}/_{296} \approx 42.2\%
\end{aligned}$$